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SIMILITUDES BETWEEN THE METHODS USED TO STUDY ENVELOPING SURFACES* I. THE PROFILES ASSOCIATED TO ROLLING CENTROIDS

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ABSTRACT

Analytical demonstrations of the similitude between the enveloping conditions forms, as they result from using the different methods to study the enveloping surfaces are exposed in this paper.

It is proved the equivalence between the methods: universal (Olivier's, Gohman's method, "minimum distance", "the family of substituting circles", "trajectory method") and particularly (the method of the normals - Willis and Nikolaev). It is also proved that, for all these methods, the condition to determine the characteristic curve for the contact between a two associated centroids in wrapping is the same one. The quality of each presented method consists of the way of expressing the wrapping, the simplicity of the calculation and also, a more suggestive presentation of the wrapping process.

1. Introduction

There are many theorems used for studing the reciprocal enveloping surfaces: the Olivier's fundamental theorem, the Gohman's kinematic method [1], [3], the Willis' method of the normals [1], [3]; and also the methods of "minimal distance" [3], [4] and [5] (Nikolaev), for helical surfaces [2], [3].

These methods are characterized by the specificity of the domain of applicability: universal, at Olivier, with a high degree of generalization, at Gohman, "the minimal distance" and "the family of substituting circles" and particular at Willis and Nikolaev.

Solving one and the same problem – the determination of the characteristic (the characteristic point) of two reciprocal envelope surfaces is permited by the specific theorems of each method.

Obviously, Olivier's theorems, with their mathematical appearance, make up the geometrical foundation of the problem, the envelope surfaces being considered as deformable linens after on one or two changeable parameters. Thus, the Olivier theorems permit the approach of any problem of envelope in geometrical but not limitative way in case of the immutable surfaces – technical surfaces, a situation that makes up the domain of application of all the other methods (metioned before).

If Gohman method represents a direct interpretation of Olivier's theorems for the immutable reciprocal surfaces, the specific theorems of the other methods present particularities of statement and forms of applicability which, formally, differentiate them.

In this paper we present the unicity of the envelope condition – a result of the different specific theorems of the before mentioned methods – as a possible demonstration of the theorems that determine the curve characteristic for known cases of the technical reciprocal envelope surfaces.

2. Reciprocal envelope surfaces associated to some rolling axoids

As it is known, the problem of the reciprocal envelope surfaces associated to some rolling axoids may be seen as a plane problem, for the current case used in technique, where

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the axoids are cylinders of revolution. This is reduced to the study of the envelope of the curves representing the sections of the surfaces with a perpendicular plane on the axis of the axoids associated to the surfaces.

This way of treating the problem of envelope by rolling has the advantage of an important simplification without reducing its generality.

Further on it is examined the unicity of the envelope condition determined by known methods, in case of engendering by rolling with a rack-tool, in order to prove that all the known theorems (Gohman, Willis, "the minimal distance", "the family of the substituting circles" and "the trajectories method") bring to the same envelope condition.

2.1. Gohman's theorem [1], [3]

We may have (fig.1) the reference systems associated to the two centroids by rolling C_1 and C_2 in case of engendering with the rack-tool:

$\xi\eta$ - the mobile system, solidary with the rack-tool;

XY – the mobile system, solidary with the centroid of the worked material;

xy - the fixed system

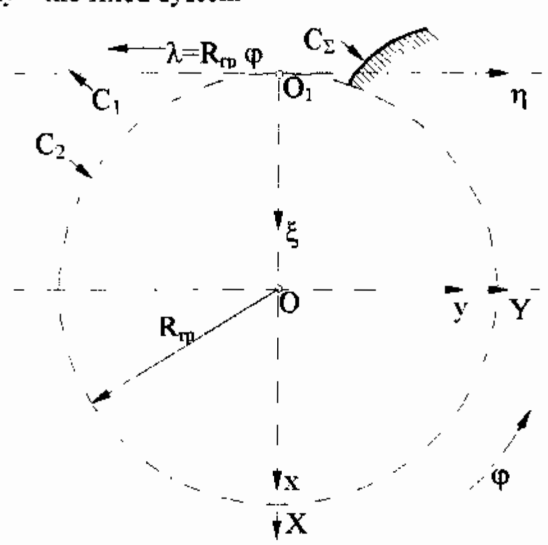

Fig.1 Reference systems

It is also defined the curve representing the perpendicular section through the surface Σ,

$$C_{\Sigma}: X = X(u);\ Y = Y(u) \qquad (1)$$

with a variable parameter u.

It is known [3] the expression of Gohman's theorem of the determination condition of envelope, expressed as:

$$\bar{N}_{\Sigma} \cdot \bar{R}_{\varphi} = 0. \qquad (2)$$

If we consider the relative movements of the reference systems as:

$$X = \omega_3(\varphi)\cdot[\xi + a];\ a = \left\| \begin{matrix} -R_{rp} \\ -R_{rp}\cdot\varphi \end{matrix} \right\| \qquad (3)$$

its "opposite":

$$\xi = \omega_3^T(\varphi)\cdot X - a \qquad (4)$$

and the definition of the vector $\vec{R}_{\varphi}$:

$$R_{\varphi} = \frac{dX}{d\varphi} = \overset{o}{\omega}_3(\varphi)\cdot[\xi + a] + \omega_3(\varphi)\cdot \overset{o}{a}_{\varphi} \qquad (5)$$

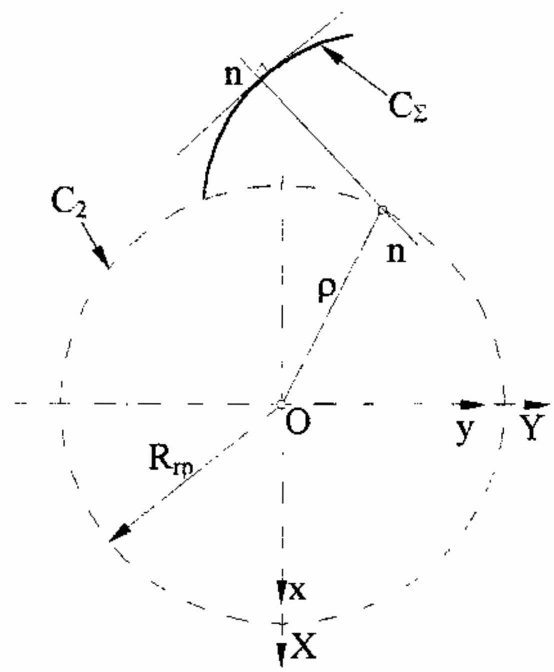

Fig.2 The normals method (Willis)

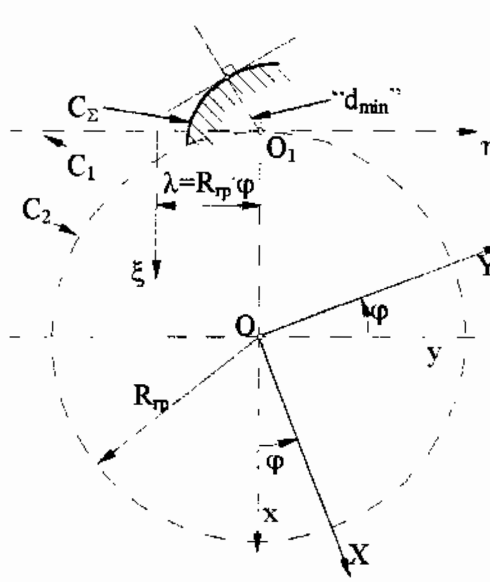

Fig.3 The theorem of „minimal distance"

where, replacing the matrix ξ with its expression (4), then it results

$$\vec{R}_\varphi = Y(u) - R_{rp} \cdot (sin\varphi) \cdot \vec{i} - \\ - [X(u) - R_{rp} \cdot cos\varphi] \cdot \vec{j}. \quad (6)$$

The normal to the curve C_Σ is expressed by

$$\vec{N}_\Sigma = Y'(u) \cdot \vec{i} - X'(u) \cdot \vec{j} \quad (7)$$

and so the envelope condition (2) becomes

$$[Y(u) - R_{rp} \cdot sin\varphi] \cdot Y'(u) + \\ + [X(u) + R_{rp} \cdot cos\varphi] \cdot X'(u) = 0. \quad (8)$$

2.2. The "Normals" theorem (Willis)

It is known [1], [3] the normals theorem for the determination of the envelope condition. For a frequent situation – a circular centroid - fig.2. associated to a curve surface that we want to determine the envelope, the normals theorem supposes that the normals to the considered profile intersect the associated centroid.

The signification of the reference systems is the same as in paragraph no. 2.1.

Defining the normal at the profile C_Σ by the equations (1):

$$[X - X(u)] \cdot X'_u + [Y - Y(u)] \cdot Y'_u = 0 \quad (9),$$

the condition of intersection with a rolling circle –C_2:.

$$X = -R_{rp} \cdot cos\varphi; \quad Y = R_{rp} \cdot sin\varphi \quad (10)$$

gives the envelope condition;

$$[R_{rp} \cdot cos\varphi + X(u)] \cdot X'_u + \\ + [-R_{rp} \cdot sin\varphi + Y(u)] \cdot Y'_u = 0. \quad (11)$$

Obviously, this condition (11), -determined by the normals theorem- is identical with the Gohman's condition (8).

2.3. The theorem of „minimal distance" [3], [4], [5]

Likewise, the rolling condition determined by the method of „minimal distance" is analyzed in fig. 3 .

We consider the same signification of the reference systems.

According to the„I-st theorem" the distance „d" is expressed by

$$d = \sqrt{\xi^2 + (\eta - R_{rp} \cdot \varphi)^2}. \quad (12)$$

The minimum of "d" supposes the condition:

$$\frac{\partial d}{\partial u} = 0 \quad (13)$$

that brings to the form

$$\xi(u) \cdot \xi'(u) + [\eta(u) - R_{rp} \cdot \varphi] \cdot \eta'(u) = 0. \quad (14)$$

Having the expression of the curve C_Σ (1) and the movement (4) that determines the family of curves

$$(C_\Sigma)_\varphi \begin{vmatrix} \xi = X(u) \cdot cos\varphi - Y(u) \cdot sin\varphi + R_{rp}; \\ \eta = X(u) \cdot sin\varphi + Y(u) \cdot cos\varphi + R_{rp} \cdot \varphi. \end{vmatrix} \quad (15)$$

from (14), it results

$$[X(u) \cdot cos\varphi - Y(u) \cdot sin\varphi + R_{rp}] \cdot [X'(u) \cdot cos\varphi - Y'(u) \cdot sin\varphi] + \\ + [X(u) \cdot sin\varphi + Y(u) \cdot cos\varphi] \cdot [X'(u) \cdot sin\varphi + Y'(u) \cdot cos\varphi] = 0 \quad (16)$$

After the development, the expression (16) is reduced to the form

$$X(u) \cdot X'(u) + Y(u) \cdot Y'(u) + \\ + R_r \cdot [X'(u) \cdot cos\varphi - Y'(u) \cdot sin\varphi] = 0, \quad (17)$$

that is identical to the expressions (8) and (11) determined by the other methods.

2.4. "The family of substituting circles" method [3], [7]

According to the "I-st theorem" of the method of "the substituting circles" (fig.4), having the family of the substitution circles

$$(C)_r \begin{vmatrix} X = -R_{rp} \cdot cos\varphi_1 - r_i \cdot cos\beta_i; \\ Y = R_{rp} \cdot sin\varphi_1 - r_i \cdot sin\beta_i, \end{vmatrix} \quad (18)$$

and the conditions of tangent of the circles of profile C_Σ (1)

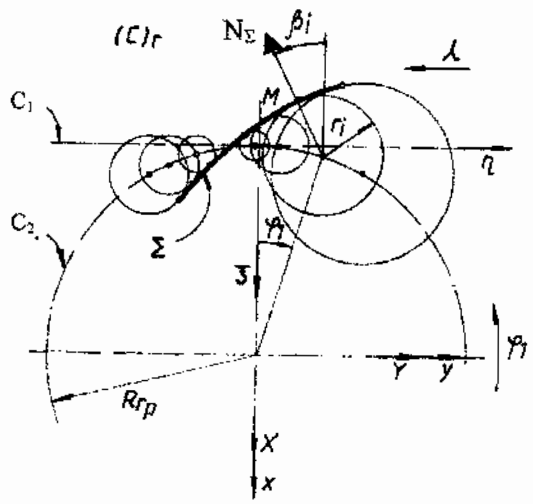

Fig.4 The family of replacing circles

$$X(u) = -R_{rp} \cdot cos\varphi_1 - r_i \cdot cos\beta_i; \\ Y(u) = R_{rp} \cdot sin\varphi_1 - r_i \cdot sin\beta_i; \\ X'(u) = r_i \cdot sin\beta_i; \\ Y'(u) = -r_i \cdot cos\beta_i, \quad (19)$$

that may be described as:

$$X(u) + R_{rp} \cdot cos\varphi = -r_i \cdot cos\beta_i; \\ Y(u) - R_{rp} \cdot sin\varphi = -r_i \cdot sin\beta_i. \quad (20)$$

We express the tangent of β_i angle:

$$tg\beta_i = \frac{Y(u) - R_{rp} \cdot sin\varphi}{X(u) + R_{rp} \cdot cos\varphi} = \frac{-X'(u)}{Y'(u)} \quad (21)$$

It results the condition of wrapping:

$$\begin{aligned}&X(u)\cdot X'(u)+Y(u)\cdot Y'(u)+\\&+[X'(u)\cdot cos\varphi-Y'(u)\cdot sin\varphi]\cdot R_{rp}=0\end{aligned} \qquad (22)$$

that is identical with the expression (8), (11) and (17) of the wrapping condition determined by the other methods.

2.5. The trajectories method [3], [8]

In case of engendering with the rack-tool, we may imagine an algorithm that permits to determinate the primary profile of the rack-tool - going from the form of the engendering surface (already done) and from a presentation of the space (the rack-tool belongs to)- in the shape of a tidy cloud of points that may be extended to all the points of the space.

We define (fig.5) the reference systems:

$\xi\eta$-a mobile system, associated to the space of rack-tool;

XY – a mobile system, associated to the worked material;

xy - a fix reference system.

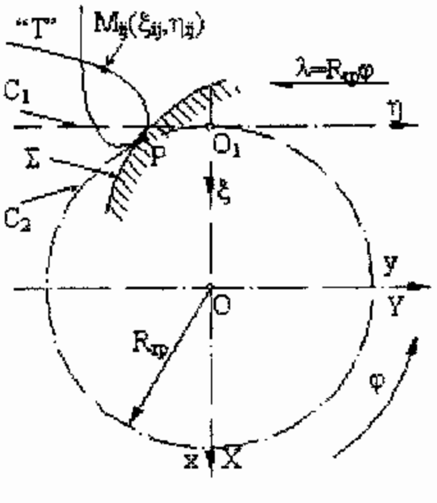

Fig.5 The relative position of the trajectory T and the profile Σ.

In the mobile reference system XY, we define the generated structure:

$$\Sigma\left|X=X(u);Y=Y(u)\right. \qquad (23)$$

with u – a variable parameter.

A certain point from the "cloud of points" $M_{ij}(\xi_{ij},\eta_{ij})$ associated to the space $\xi\eta$, in the relative movement towards the mobile system XY,

$$X=\omega_3(\varphi)[\xi+a] \qquad (24)$$

$$a=\left\|-R_{rp} \quad -R_{rp}\cdot\varphi\right\|$$

describes a trajectory, defined by the equations:

$$\begin{Vmatrix}X\\Y\end{Vmatrix}=\begin{Vmatrix}cos\varphi & sin\varphi\\-sin\varphi & cos\varphi\end{Vmatrix}\cdot\left[\begin{Vmatrix}\xi_{ij}\\\eta_{ij}\end{Vmatrix}+\begin{Vmatrix}-R_{rp}\\-R_{rp}\cdot\varphi\end{Vmatrix}\right] \qquad (25)$$

or

$$T\left|\begin{aligned}&X=[\xi_{ij}-R_{rp}]\cdot cos\varphi+[\eta_{ij}-R_{rp}\cdot\varphi]\cdot sin\varphi;\\&Y=-[\xi_{ij}-R_{rp}]\cdot sin\varphi+[\eta_{ij}-R_{rp}\cdot\varphi]\cdot cos\varphi.\end{aligned}\right. \qquad (26)$$

The equation (26) defines a cycloidal curve associated to the circle with R_{rp} ray.

The condition that the point $M_{ij}(\xi_{ij},\eta_{ij})$ of the „cloud of points" belongs at the same time to a trajectory „T" and has only one contact point with the profile Σ, brings to the identification equations:

$$\begin{aligned}&X'(u)=\xi_{ij}\cdot sin\varphi+[\eta_{ij}-R_{rp}\cdot\varphi]\cdot cos\varphi;\\&Y'(u)=-\xi_{ij}\cdot cos\varphi-[\eta_{ij}-R_{rp}\cdot\varphi]\cdot sin\varphi.\end{aligned} \qquad (27)$$

(see fig.5).

Using (27) we may express the dependence between the coordinates of the current point of the cloud and the steering parameters of the tangent to the engendering profile:

$$\begin{aligned}&\xi_{ij}=-\left[X'(u)\cdot sin\varphi+Y'(u)\cdot cos\varphi\right];\\&\eta_{ij}=X'(u)\cdot cos\varphi-Y'(u)\cdot sin\varphi+R_{rp}\cdot\varphi.\end{aligned} \qquad (28)$$

Also, from (27) and (28) it results:

$$X(u)X'(u)+Y(u)Y'(u)=-[\eta_{ij}-R_{rp}\cdot\varphi]\cdot R_{rp} \qquad (29)$$

or, using the expression (28), we may get to the form:

$$\begin{aligned}&X(u)X'(u)+Y(u)Y'(u)=\\&=R_{rp}\left[-X'(u)cos(\varphi)+Y'(u)sin(\varphi)\right]\end{aligned} \qquad (30)$$

The condition (30) represents the well-known form of the wrapping condition specific for all the known methods: Gohman's method, Willis (the normals theorem), the method of the „minimal distance", the „family of substituting circles" method.

Extending the equations (28) and (30) to the whole space of the tool, we get the envelope of the profile Σ- the primary periferical surface S of the rack- tool.

$$S\left|\begin{aligned}&\xi=X(u)\cdot cos\varphi-Y(u)\cdot sin\varphi+R_{rp};\\&\eta=Y(u)\cdot sin\varphi+X(u)\cdot cos\varphi+R_{rp}\cdot\varphi\end{aligned}\right. \qquad (31)$$

and

$$X\cdot X'+Y\cdot Y'=R_{rp}\cdot(-X'\cdot cos\varphi+Y'\cdot sin\varphi). \qquad (32)$$

Eliminating, for example, the parameter „u", the profile „S" is expressed in the form:

$$S\left|\begin{aligned}&\xi=\xi(\varphi);\\&\eta=\eta(\varphi).\end{aligned}\right. \qquad (33)$$

This way, the ensemble of equations (33) determines the wrapping of the profile Σ (representing the primary periferical surface of the rack-tool) as the geometrical place of the points $M_{ij}(\xi_{ij},\eta_{ij})$ belonging to the cloud of points associated to the space $\xi\eta$, for that the relative trajectories towards the space XY in the rolling movement of the two associated centroids are tangents at the generated profile.

2.6. The method of cycloidal trajectories [9], [10]

To solve the problem of the profiles in wrapping associated to a centroids couple in rolling mouvement we may use a new formulation of the fundamental theorem of gearing (see also fig.6 – the method of "cycloidal trajectories"

In the rolling movement of the two centroids (where the family of profiles Σ is associated) the current point of this profile describes in the space of the associated centroid (in fig. C_1) a family of cycloidal curves that we may determined analytically the envelope.

The envelope of a profile associated to a centroid and belonging to a cuple of rolling centroids is the envelope of the family of trajectories described by the points of the profile in the space associated to the conjugated centroid.

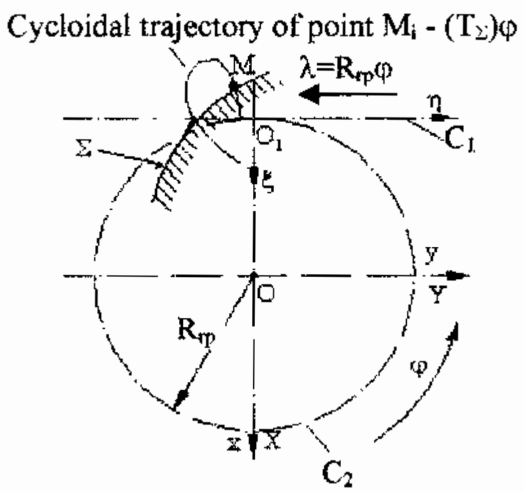

Fig.6 Cycloidal trajectories

If the defined references systems as in fig.6 are accepted, the family of „cicloydal trajectories" is generated by the relative movement of the profile Σ, defined by (1), towards the system ξηζ :

$$(T_\Sigma)_\varphi \begin{cases} \xi = X(u)\cos\varphi - Y(u)\sin\varphi + R_{rp}\ ; \\ \eta = X(u)\sin\varphi + Y(u)\cos\varphi + R_{rp}\varphi. \end{cases} \quad (34)$$

The envelope of this family of trajectories is the profile associated to the centroid C_2 and conjugated to the profile Σ.

The geometrical condition for wrapping the family of cycloidal curves (1) is

$$\frac{\xi_u'}{\xi_\varphi'} = \frac{\eta_u'}{\eta_\varphi'} \quad (35)$$

where $\xi_u', \xi_\varphi', \eta_u', \eta_\varphi'$ are partial derivated from the equations of the family of curves (1).

Theoretically, these derivated may be written in form:

$$\begin{aligned} \xi_u' &= X_u' \cos\varphi - Y_u' \sin\varphi\ ; \\ \eta_u' &= X_u' \sin\varphi + Y_u' \cos\varphi\ ; \\ \xi_p' &= -X(u)\sin\varphi - Y(u)\cos\varphi\ ; \\ \eta_p' &= X(u)\cos\varphi - Y(u)\sin\varphi + R_{rp}. \end{aligned} \quad (36)$$

Processing (35) and (36) it results the condition:

$$-X(u)X_u' + Y(u)Y_u' = -R_{rp}\left(-X_u' \cos\varphi + Y_u' \sin\varphi\right) \quad (37)$$

that is identical with the other presented forms, so we have again the equivalence of the wrapping conditions.

3. Conclusions

It's obviously from the previous presentation that the wrapping condition – the curve surfaces associated to some axoids (centroids) in rolling movement - is the same for all the examinated situations.

The specific way of expressing the theorem of wrapping, in the different methods: Gohman, Willis, „minimal distance", substituting circles", „trajectories method" – brings finally to the same analytical expression of the wrapping condition. The way of expressing the wrapping, the simplicity of the calculation and also a more suggestive presentation of the wrapping process are the attributes of each presented methods.

Bibliography

[1] **Litvin, F., L.** - *Theory of gearing*, NASA, Reference Publication, Washington, D.C., 1984:

[2] **Linksin, V., S.** - *Teoria vintovah poverhnostei*, Masinostroenie, 1968;

[3] **Oancea, N.** – *Metode numerice pentru profilarea sculelor*, Universitatea din Galati, vol. I-VIII, 1990-2000;

[4] **Oancea, N.** - *A new method in profiling of the tools which worek by wrapping up.* In: Techica Italiana, nr.2, 1992, p.85-94;

[5] **Oancea, N., Oancea, V.** - *Geometrical design of Cutting Tools with Surfaces of Revolution.* In Procedings of the Institution of Mechanical Engineers, vol.221, part C, p.559-566;

[6] **Oancea, N., Neagu, M.** - *A New Method for Tools Profiling for Helical Surface Generation.* In Proccdings ITC2000, X-th International Conference on Tools, MISCOLC, 2000, p.89-98;

[7] **Oancea, N** - *The substituting circles method, a new expression of the theory in wrapping surfaces. The base theory.* In : Constructia de Masini, Anul 50, Supliment nr. 2, 1998, p.11-15;
[8] **Oancea, N., Neagu, M., Fetecau, C.** - *The trajectories method, a new method of specializing the tools that processes by wrapping.* In: Constructia de Masini, anul 52, nr.3-4, 2000, p.29-34;
[9] **Oancea, N., Teodor, V.** – *The cycloid trajectory method for wrapping surfaces associated of rolling centroid study. I. Algoritm.* In: Analele Universitatii Dunarea de Jos din Galati, anul 2001, Fascicula V, p.5-9.
[10] **Teodor, V., Oancea, N.** – *The cycloid trajectory method for wrapping surfaces associated of rolling centroid study. II. Applications.* In: Analele Universitatii Dunarea de Jos din Galati, anul 2001, Fascicula V, p.10-17.

SIMILITUDINI INTRE METODELE UTILIZATE IN STUDIUL SUPRAFETELOR IN INFASURARE
I.PROFILE ASOCIATE UNOR CENTROIDE IN RULARE

REZUMAT

Demonstrarea analitica a similitudinii intre diferite procedee de infasurare rezulta folosind diverse metode de studiu a infasurarii suprafetelor prezentate in aceasta lucrare.

Este dovedita o echivalenta intre diferite metode: universala (Olivier, Gohman, distanta minima, familia de cercuri de substituire, metoda traiectoriilor) si particulara (metoda normalelor – Willis si Nikolaev). De asemenea, este dovedit pentru toate aceste metode, conditia de determinare a curbei caracteristice la contactul a doua centroide asociate in infasurare. Fiecare metoda prezentata se caracterizeaza printr-o simplitate a calculului si, de asemenea, printr-o sugestiva reprezentare a procesului de infasurare.

SIMILITUDES ENTRE LES MÉTHODES ÉTUDIAIT ENVELOPPER DES SURFACES
I. LES PROFILS ASSOCIÉS AUX CENTRES DE SURFACE DE ROULEMENT

ABSTRAIT

Les démonstrations analytiques du similitude entre l'enveloppement conditionne des formes, car elles résultent d'employer les différentes méthodes pour étudier les surfaces d'enveloppement sont exposées en cet article.

On le prouve l'équivalence entre les méthodes: universel (méthode d'Olivier.s, de Gohman.s, distance de minimum, famille de the de substituer des cercles, méthode de trajectory.) et en particulier (la méthode de normals- Willis et Nikolaev). On le montre également que, pour toutes ces méthodes, la condition pour déterminer la courbe caractéristique pour le contact entre deux centres de surface associés dans l'emballage est la même. La qualité de chaque méthode présentée comprend la voie d'exprimer l'emballage, la simplicité du calcul et aussi, une présentation plus suggestive du processus d' emballage.

SIMILITUDES ZWISCHEN DEN METHODEN VERWENDETE, DAS EINSCHLAGEN DER OBERFLÄCHEN ZU STUDIEREN
I. DIE PROFILE DAZUGEHÖRIG ZU DEN ROLLENCSCHWERPUNKTEN

ABSTRAKTE

Analytische Demonstrationen des similitude zwischen dem Einschlagen bedingt Formulare, da sie aus dem Verwenden der unterschiedlichen Methoden, um die Einschlagenoberflächen zu studieren werden herausgestellt in diesem Papier resultieren.

Es wird der Gleichwertigkeit zwischen den Methoden nachgewiesen: Universalität (Olivier.s, Gohman.s-Methode, minimumabstand, thefamilie des Ersetzens der Kreise, trajectorymethode.) und besonders (die Methode des normals- Willis und Nikolaev). Es wird auch nachgewiesen, daß, für alle diese Methoden, die Bedingung zum Feststellen der Kennlinie für den Kontakt zwischen zwei dazugehörige Schwerpunkte, bei der Verpackung die gleiche ist. Die Qualität jeder dargestellten Methode besteht aus der Weise des Ausdrückens der Verpackung, der Einfachheit der Berechnung und auch, eine andeutendere Darstellung des aufwickelnprozesses.